## FIRST PREBOARD EXAMINATION (2020-21) <br> CLASS: X

Subject: MATHEMATICS (STANDARD-041)
Time Allowed: 3 Hours

Date: 04.01.2021
Maximum Marks: 80

General instructions:

1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.
3. Please check that this question paper contains 12 printed pages.

Part-A :

1. It consists of two sections- I and II.
2. Section I has 16 questions of 1 mark each. Internal choice is provided in 5 questions.
3. Section II has 4 questions on case study. Each question has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part - B :

1. It consists of three sections- III, IV and V.
2. Question No 21 to 26 are Very short Answer Type questions of 2 marks each.
3. Question No 27 to 33 are Short Answer Type questions of 3 marks each.
4. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
5. There is no overall choice. However, internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks. You have to attempt only one of the alternatives in all such questions.

## PART - A

## SECTION - I

Section I has 16 questions of 1 mark each. Internal choice is provided in 5 questions.

1. There is a circular path around a sports field. Sanhita takes 24 minutes to drive one round of the field, while Amit takes 16 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

## OR

The decimal expansion of $\frac{343}{2^{3} \times 5^{2} \times 7^{3}}$ will terminate after how many decimal places?
2. If $\alpha$ and $\beta$ are the zeroes of the polynomial $x^{2}+3 x-2$, then find the value of $\alpha^{2} \beta^{3}+\alpha^{3} \beta^{2}$ ?
3. For which values of $p$ does the pair of equations $4 x+p y+8=0$ and $2 x+2 y+2=0$ has unique solution?
4. If $(x, y)$ is the solution of the linear equations $2 x+3 y=11$ and $2 x-4 y+24=0$, then find the values of $x$ and $y$.
5. If the roots of the equation $(a-b) x^{2}+(b-c) x+(c-a)=0$ are equal, then find the value of $b+c$ in terms of ' $a$ '.
6. If the roots of the quadratic equation $3 x^{2}+7 x-k=0$ are reciprocals of each other, then find the value of ' $k$ '.

## OR

If $\alpha$ and $\beta$ are the roots of a quadratic equation such that $\alpha+\beta=2$ and $\alpha \beta=3$, then find the quadratic equation.
7. Find the $20^{\text {th }}$ term from the last term of the $\mathrm{AP}: 3,8,13, \ldots . .253$.

## OR

Find the value of $x$ for which $(8 x+4),(6 x-2)$ and $(2 x+7)$ are in AP.
8. In the given figure, if $\angle \mathrm{A}=90^{\circ}, \angle \mathrm{B}=90^{\circ}, \mathrm{OB}=4.5 \mathrm{~cm}, \mathrm{OA}=6 \mathrm{~cm}$ and $\mathrm{AP}=4 \mathrm{~cm}$, then find the value of QB ?

9. In $\triangle A B C$, if $\sqrt{3} \tan \mathrm{~A}=2 \cos \mathrm{~B}=1$, then find the value of angle C ?
10. Evaluate: $\frac{\sin 45^{\circ}}{\cos 45^{0}}+\frac{1}{\operatorname{cosec} 30^{0}}$
11.In the given figure, a circle is inscribed in a quadrilateral $A B C D$ in which $\angle B=90^{\circ}$. If $A D=23 \mathrm{~cm}, A B=29 \mathrm{~cm}$ and $D S=5 \mathrm{~cm}$, find the radius $(\mathrm{r})$ of the circle.


## OR

In the figure, PT is tangent to a circle with centre $\mathrm{O}, \mathrm{PT}=36 \mathrm{~cm}$, $\mathrm{AP}=24 \mathrm{~cm}$. Find the radius of the circle.

12.To draw a pair of tangents to a circle which are inclined to each other at an angle of $50^{\circ}$, it is required to draw tangents at the end points of two corresponding radii of the circle. Find the angle between the radii.
13. If the circumference and the area of a circle are numerically equal, then find the diameter of the circle?
14.If the heights of two cylinders are in the ratio $4: 3$ and their radii are in the ratio $2: 3$, then find the ratio of their volumes.
15.Find median of the data, using empirical relation when it is given that mode $=12.4$ and mean $=10.5$ .
16.A card is drawn from a pack of cards numbered from 1 to 100 . Find the probability that the number on the card is a perfect cube?

## OR

If the probability of winning a game is 0.892 , then find the probability of losing the game?

## SECTION - II

Case study based questions are compulsory. Attempt any four sub-parts of each question. Each sub-part carries 1 mark.
17. Case study based - 1

There are two cottage industries A and B having the same number of employees. In industry A, It was observed on a particular day the cost of production of each toy (in rupees) was 3 more than twice the number of toys produced on that day. If the total cost of production on that day was ₹90, find the number of toys produced. In industry B, the cost of production of each toy (in rupees) was found to be 6 more than four times the number of toys produced on that day. On a particular day the total cost of production was ₹550.


Using the above information answer the following case study questions.
(a) If the number of toys produced in A be ' $x$ ' then cost of production of each toy is (in rupees )
(A) $2 x+3$
(B) $3 x+2$
(C) $x(2 x+3)$
(D) $2 x^{2}+3$
(b) The number of toys produced in industry A on that particular day is
(A) 2
(B) 3
(C) 6
(D) 5
(c) If the number of toys produced in B be ' $y$ ' then, cost of production of each toy is (in rupees )
(A) $y(4 y+6)$
(B) $4 y+6$
(C) $y(6 y+4)$
(D) $4 y^{2}+6$
(d)The number of toys produced in industry B on that particular day is
(A) 10
(B) 12
(C) 11
(D) 4
(e) The number of toys produced in industry B than industry A on that particular day is
(A) 4
(B) 5
(C) 6
(D) 7
18. Case study based - 2

The students of a school can stand in two ways, in circles or in row. In case of circles: there are 23 students in the first circle, 21 in the second, 19 in the third and so on. There are 5 students in the last inner most circle. How many circles are there? In case of rows: 20 students are there in the first row, 19 in the second row, 18 in the next row and so on. If 5 students are there in the top most row. It was observed that students were talking with one another when they were made to stand in the circular pattern.
Therefore, the teacher chose to make them stand in rows.

(a) If the students are standing in circles then the number of students standing in $5^{\text {th }}$ circle is
(A) 17
(B) 16
(C) 15
(D) 14
(b) If the students are standing in circles then the number of circles is
(A) 5
(B) 10
(C) 15
(D) 20
(c) If the students are standing in rows then the number of students standing in $7^{\text {th }}$ row is
(A) 14
(B) 16
(C) 15
(D) 13
(d) If the students are standing in rows then the number of rows is
(A) 14
(B) 15
(C) 16
(D) 17
(e) Total number of students standing in rows is
(A) 100
(B) 200
(C) 150
(D) 250
19. Case study based - 3

Ant is a very hardworking insect. It can lift many times heavier weight than its own weight. An ant comes out of its hole, which is at $A(4,2)$. It moves along $A B$ and reach at $B(-5,1)$.From point $B$, it observed that a piece of grain is lying at point $C(3,8)$. So, it moves along $B C$ to reach the point C form C , it picks the grain and moves along CA to reach the hole at A.

(a) The distance travelled by ant from A to B is ------------Units
(A) $\sqrt{82}$
(B) $\sqrt{113}$
(C) $\sqrt{73}$
(D) $\sqrt{37}$
(b) The distance travelled by ant from $B$ to $C$ is $\qquad$
(A) $\sqrt{82}$
(B) $\sqrt{113}$
(C) $\sqrt{73}$
(D) $\sqrt{37}$
(c) The distance travelled by ant from C to A is

(A) $\sqrt{82}$
(B) $\sqrt{113}$
(C) $\sqrt{73}$
(D) $\sqrt{37}$
(d) In all how much distance (in Units) did the ant cover (approx.)?
(A) 20.67
(B) 25.76
(C) 23.76
(D)24.67
(e) The centroid of triangle $A B C$ is $\qquad$
(A) $(1,3)$
(B) $\left(\frac{2}{3}, \frac{7}{3}\right)$
(C) $\left(\frac{2}{3}, \frac{11}{3}\right)$
(D) $\left(\frac{11}{3}, \frac{2}{3}\right)$
20. Case study based - 4

A boy standing on a horizontal plane finds a bird falling at a distance of 100 m from him at an elevation of $30^{\circ}$. A girl standing on the roof of a 20 m high building finds the angle of elevation of the same bird at the same time to be $45^{\circ}$. Both boy and girl are on opposite sides of the bird and the girl wants to save the bird.

(a) The height of the Bird from the horizontal plane $(\mathrm{AB})$ is ------meters.
(A) 25
(B) 50
(C) 75
(D) 100
(b)The horizontal distance of the Bird from the Boy (BC) is ------meters.
(A) $25 \sqrt{3}$
(B) $50 \sqrt{3}$
(C) $75 \sqrt{3}$
(D) $100 \sqrt{3}$
(c) The horizontal distance of the Bird from the Girl (DE) is ------meters.
(A) 20
(B) 30
(C) 40
(D) 45
(d)The distance of the Bird from the Girl (AE) is ------meters.
(A) $20 \sqrt{2}$
(B) $25 \sqrt{3}$
(C) $30 \sqrt{2}$
(D) $30 \sqrt{3}$
(e) The horizontal distance between the Boy and the Girl is------meters. (Take $\sqrt{3}=1.73$ )
(A) 106.5
(B) 110.5
(C) 112.5
(D) 116.5

## PART - B

All questions are compulsory. In case of internal choices, attempt any one.

$(6 \times 2=12)$
21.The HCF and LCM of two numbers are 4 and 9696 respectively. If one number is 96 , find the other number.
22. $\alpha$ and $\beta$ are the zeroes of the quadratic polynomial $x^{2}-7 x+a$. Find the value of ' $a$ ', if $3 \alpha+4 \beta=24$.
23.Three vertices of a parallelogram taken in order are $(-1,-6),(2,-5)$ and $(7,2)$ . Find its fourth vertex.

## OR

' P ' is the midpoint of the line segment joining the points $\mathrm{A}(2,1)$ and $B(5,-8)$. If the point $P$ lies on the line $2 x-y+k=0$, then find the value of ' $k$ '.
24. Prove that,

$$
\begin{gathered}
3(\operatorname{Sin} \theta-\operatorname{Cos} \theta)^{4}+6(\operatorname{Sin} \theta+\operatorname{Cos} \theta)^{2}+4\left(\sin ^{6} \theta+\cos ^{6} \theta\right)=13 \\
\text { OR }
\end{gathered}
$$

Prove that, $\frac{\tan x}{1-\cot x}+\frac{\cot x}{1-\tan x}=1+\sec x \cdot \operatorname{cosec} x$
25. Draw a pair of tangents to a circle of radius 3 cm from a point P which is 8 cm away from the centre of the circle.
26. A survey regarding the heights (in cm ) of 50 girls of a class was conducted and the following data was obtained. Find the mode of the given data.

| Height(in cm) | $120-130$ | $130-140$ | $140-150$ | $150-160$ | $160-170$ | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of girls | 2 | 8 | 12 | 20 | 8 | $\mathbf{5 0}$ |

## SECTION - IV

(7 X $3=21$ )
27.A merchant has 120 litres of oil of one kind, 180 litres of another kind and 240 litres of third kind. He wants to sell the oil by filling the three kinds of oil in tins of equal capacity. What should be the greatest capacity of such a tin?

## OR

If the HCF of 210 and 55 is expressible in the form of $210 \times 5+55 y$, find $y$.
28.Find the zeroes of the polynomial $x^{2}+7 x+12$ and verify the relation between the zeros and its coefficients.
29. Prove that, in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

## OR

$B L$ and $C M$ are the medians of $\triangle \mathrm{ABC}$ right angled at A . Prove that $4\left(B L^{2}+C M^{2}\right)=5 B C^{2}$


Page 10 of 12
30.The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are $30^{\circ}$ and $45^{\circ}$ respectively. Find the height of the multi-storeyed building and the distance between the two buildings.

31. PQ is a chord of length 8 cm of a circle of radius 5 cm . The tangents at $P$ and $Q$ intersect at a point T. Find the length TP.

32.In the given figure, arcs have been drawn with radii 10 cm each and with centres P, Q and R. Find the area of the shaded region. (Take $\pi=3.14$ )

33. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting
(i) a red card
(ii) neither a spade nor a king
(iii) either a queen or a diamond
34. Selvin's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump (underground tank), which is in the shape of a cuboid. The sump has dimensions $1.57 \mathrm{~m} \times 1.44 \mathrm{~m} \times 0.95 \mathrm{~m}$. The overhead tank has its radius of 60 cm and its height is 95 cm . Find the height of the water left in the sump after the overhead tank has been completely filled with water from a sump which had been full. Compare the capacity of the tank with that of the sump. (Take $\pi=3.14$ )

## OR

A right cylindrical container of radius 6 cm and height 15 cm if full of icecream, which has to be distributed to 10 children in equal cones having hemispherical shape on the top. If the height of the conical portion is four times its base radius, find the radius of the ice-cream cone.
35. A boat covers 32 km upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of the boat in still water and that of the stream.
36. For the following distribution, compute the mean and median by using the formula.

| Class | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ | $60-70$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 5 | 15 | 20 | 23 | 17 | 11 | 9 |

